What Constitutes Quantitative Literacy for Pre-service Primary Teachers?

ROBERT PEARD

Queensland University of Technology

Abstract

The author coordinates a Foundations Unit, Quantitative Literacy at Queensland University of Technology for pre-service BEd (primary) students of whom there are about 350 each year. There is no one single definition of or universal agreement on what constitutes quantitative literacy. However one major component requires the students to constantly question their beliefs and understanding of quantitative (mathematical and scientific) ideas. We ask "Why is it that in many school mathematics and science classrooms so many students suspend their beliefs and thinking?" Why are beliefs without foundation, misconceptions and prejudices so common? In the unit students examine the reasoning behind induction, deduction, hypothesis formation and the development of scientific theories. This paper examines the issues raised in the unit including: the nature of quantitative knowledge, and the bases of many of our beliefs, mathematical, scientific and everyday.

The Purpose of this Study

Concern with primary teachers' subject knowledge in mathematics and science has been extensively documented in the literature over the past two decades: in mathematics education, for example by Ball (1990); Frid, Goos & Sparrow (2006); Peard (2001); Relich & Way (1992); White, Way, Perry & Southwell (2006); Ryan & McCrae (2006) and in science education for example by Neal, Smith & Johnson (1990) and Taylor & Francis (2001).

White et al. (2006) tested 78 pre-service teachers' mathematical abilities and reported that "overall achievement was poor" (p. 43). Attempts to improve pre-service teachers' subject knowledge have been varied. In New South Wales, for example, a prerequisite of Year 12 mathematics for teacher accreditation has been placed. However, White et al (2006) question the effectiveness of this noting that almost all the participants in their study met this requirement. Furthermore, an earlier study by the author of the present paper found students at QUT who had done only Year 10 mathematics achieved little differently in the Foundations unit from those who had done a Year 12 mathematics subject. Furthermore, there was little difference in achievement between those students who had done an academic mathematics subject to those who had done a non-academic subject and concluded "the selection of an academic course in Year 12 does not in itself mean a greater chance of success at the tertiary level, at least in primary teacher education" (Peard, 2004, p.424). In addition, there is evidence, both anecdotal and research, that many students begin teacher education displaying misconceptions, negative attitudes towards and apprehension of both mathematics and science (See, for example, Frid, Goos & Sparrow, 2006; Grootenboer & Lowrie, 2002; Kruger & Summers, 1998; White, Way, Perry & Southwell, 2006). Ryan & McCrea (2006, p. 87) reported "significant proportions of cohorts on entry to initial teacher education have the same errors, misconceptions and incorrect strategies (in mathematics) as children." Kruger & Summers (1988) reported that primary teachers' misconceptions were common and over a decade later Taylor & Francis (2001) observed that teachers as well as children have misconceptions about primary science topics.

There is general agreement from the findings of research into pre-service teachers' beliefs that these students' misconceptions are acquired during their school experiences (Kane, Sandretto, & Heath, 2002). There is also evidence that negative attitudes contribute to poor classroom teaching which in turn contributes to poor pupil attitudes, beliefs and performance outcome (White, et al., p. 36). Informal and anecdotal evidence gathered by the author during the implementation of the Foundations unit confirm these observations and are reported in this paper. White et al. (2006) support this author's view that "if these pupils go on to become teachers, a cycle of negativity may be created unless an appropriate intervention breaks the cycle" (p. 36). Perry et al. (2005, p. 631) confirm the consistent correlation of negative attitudes towards mathematics and achievement in the subject.

Ryan & McCrea (2006, p. 87) believe that it is the responsibility of the tertiary institute to make the content comprehensible to the student. White et al. (2006, p. 47) note that the best way to reach the required level of subject knowledge is via well constructed units in teacher education programs. Hence, it would appear that if change is to occur, it must come through suitable intervention at the tertiary level. However, most pre-service primary teacher education programs in Australia are able to allow only a limited time for the teaching of mathematical and scientific content. It is therefore important to make the most efficient use of the limited time available to improve the general mathematical and scientific competencies of pre-service primary teachers. In order to do this, we must first establish what constitutes quantitative literacy and how it can be best taught.

This paper will outline an attempt at appropriate intervention by the implementation of the integrated unit, Quantitative Literacy, which has been developed by the author. The unit consists of an integration of topics from mathematics and science. Objectives of this unit include the development of an understanding of the nature of the disciplines, the construction and acquisition of knowledge and the formation and recognition of the bases for beliefs in the disciplines.

Dimensions of the Unit

There is no one single definition of or universal agreement on what constitutes quantitative literacy. Clearly it includes "numeracy". However attempts at defining even this simpler term have been fraught with difficulty (Willis, 1989). The authors of "A National Statement on Mathematics for Australian Schools" (Australian Education Council, 1991) made the point that while the desirable characteristics of a numerate person can be identified; it is much more difficult to say precisely what numeracy is.

Willis (1989) in recognising the inadequacy of any single definition goes on to say:

> A numerate person would use a blend of mathematical, contextual and strategic knowledge when required to use mathematics in a practical setting. (p. 34)

Kemp & Hogan (2000), building on the ideas of Willis attempt to define numeracy:

> Numeracy is having the disposition and critical ability to choose and use appropriate mathematical knowledge strategically in specific contexts.

The unit begins by recognizing the importance of quantitative literacy. As well known Canadian scientist David Suzuki points out:

> Today the most powerful force affecting our lives is not politics, business, celebrity or sports despite the coverage they receive in the media. By far the greatest factor shaping the world is science….Without a basic knowledge of scientific terms and concepts and an understanding of how science differs from other ways of knowledge we cannot find real solutions to such issues as global warming, toxic pollution, species extinction, overpopulation, alienation and drug abuse. (Suzuki, 2006, p. 324)

This leads us to ask what constitutes "basic knowledge of scientific concepts?" In terms of scientific content, when we consider what is "essential", what is "important" and what is "desirable", there is simply too much factual information for any educators to come to agreement. Rather we need to consider how scientific knowledge is acquired. Hence a major component of the unit is the requirement of students to constantly question their beliefs and understanding of quantitative (mathematical and scientific) ideas. It is the opinion of the present author that in order to develop effective quantitative literacy negative attitudes and perceptions reported in the literature must first be overcome and that this is best accomplished by:

- Recognising that there is a vast amount of "factual" content and not all of it can be or needs to be considered. Rather we must question what content is important and how knowledge about it is established.
- Challenging established beliefs
- Presenting all subject content in relevant social contexts

In order to do this we identify three main dimensions of the unit:

- The learning of selected scientific and mathematical concepts and skills, that is, content topics and the development of these in response to social need. Recognising that the body of knowledge is vast, we must select relevant topics in order to:
- Learn about science and mathematics in a relevant context. The context of the subjects refers to the historical, social and cultural aspects. There is a common misconception that mathematics and science are culture free subjects. Bishop (1998) gives a thorough refutation of this notion. In this unit, mathematical and scientific thinking, beliefs, and problem solving are examined only in social contexts.
- Students of quantitative literacy must learn about themselves as learners, that is to say that there is a meta-learning dimension to the Unit.

Within these dimensions we examine:

- The acquisition of mathematical and scientific knowledge. Beliefs about the nature of mathematics, its roles in society, and the contribution it has made to the growth of knowledge. Myths and misconceptions about mathematics and science.
- The development of knowledge in response to social need. The scientific method and the formation of hypotheses. There is a focus on challenging existing beliefs and justifying personal beliefs.
- The nature and role of problem solving. Induction and deduction in mathematics and science. The role of patterning and generalisations in the formation of hypotheses.

Delivery of the Unit

The unit is delivered as a large group (300+) lecture of 1-2 hours followed by a small group (25) tutorial of two hours. Students keep a reflective journal of their reaction to the lecture and tutorial activities. Journal entries are made each week. These consist of answers to activity questions from the tutorial and a reflection on the lecture topics. Assessment criteria of journals include evidence of reflection on and personal reaction to the lecture topics. This encourages honest responses when asked, for example, to answer in their journal "What did you learn from the lecture? What parts of the lecture did you find most interesting?" As a result of many such entries the prevalence of misconceptions becomes apparent.

Throughout this unit we stress that much quantitative knowledge has been constructed in response to social need. Social needs have included:

- counting and measuring quantities
- computing
- measuring time

- measuring position: astronomy and navigation
- measuring chance
- analysing data, and problem solving in these contexts.

The unit begins in the first week with an examination of students' school experiences and their beliefs. We consider why is it that in many classrooms some students "suspend their beliefs" and thinking - that is content is not meaningful to them?

Questions asked include those of the type:

- A flight from Brisbane to Sydney with 200 passengers takes about 1 hour to complete the journey. How long will a flight take if it has 300 passengers?
- A class has 14 boys and 12 girls. How old is the teacher?

The nature of quantitative knowledge and the basis of beliefs follow in week 2. It is at this stage that we challenge many of the students' conceptions of what is mathematics? Journal entries by students consistently report that they view mathematics as a meaningless set of rules and procedures to be learned and followed and scientific knowledge as a set of facts to be memorized. Knowledge constructed in response to social need is then examined from the following topics. Throughout, a problem solving approach is taken.

- the measurement of time and the history of the calendar
- the measurement of position and the earth in space
- number systems and the history of our system
- measurement of quantities and metric measurement
- the measurement of chance
- interpretation of data
- geometry in the world: nature, art, architecture

Common Misconceptions

From an examination of journal entries over a number of years, common views and misconceptions of mathematics reported include:

- Only very intelligent people can understand it. Some people can't do it at all.
- Mathematics never changes.
- Mathematics requires the memorisation of lots of rules and formulae.
- There's no room for opinions in mathematics, everything is either right or wrong, true or false.

Misconceptions in science reported in the literature (Taylor & Francis, 2001) are also evidenced here. These include:

- The earth is closer to the sun in summer (common),
- The sun is always directly overhead at 12:00 noon.
- The phases of the moon are caused by shadows cast on its surface by other objects in the solar system.
- The moon emits its own light (rare)
- The moon has a dark side
- The terms AD and BC have been in use since 1AD (526 AD was the first year)

Of the moon misconceptions, it is interesting to note that many who held these views had completed successfully advanced senior mathematics. It is quite disturbing that these same students would have studied extensive calculus. Calculus was developed by Newton, largely to enable him to explain planetary motion and the functioning of the physical world. Yet here we have students studying calculus who have little comprehension of that very physical world. We might ask how widespread is this phenomenon and what does it imply about the whole nature of senior secondary education. This is included in the conclusion as a recommendation for further research.

Some Bases for our Beliefs and Reasoning

When asked to write about the basis for their mathematical and scientific beliefs the vast majority answer: the authority of the teacher and the text books. Students are generally unaware that many of our beliefs, mathematical, scientific and everyday have different bases; authority, personal beliefs and faiths, beliefs arrived at inductively and deductively, and beliefs without foundation.

We recognise the need for beliefs based on authority; young children must rely on their parents authority as to what is good for them, how to cross the road, what to eat, etc. Pupils rely on the authority of teachers and texts. Citizens rely on the authority of politicians, the media, newspapers, magazines, books etc. However, not all of these authorities are always reliable. Most of our students are surprised to hear that many of the things they learned from an “authority” are in fact totally wrong. The most common of these is the misconception that Columbus was the first to show that the earth is round. Most are unaware that the Greeks not only knew the shape of the earth some 2000 years earlier but also that Eratosthenes (c. 200BC) had accurately measured the circumference. The facts that you can see further from higher elevations, that as a ship disappears over the horizon its sail disappears last, and the earth’s shadow on the moon during an eclipse are all observable evidence of this and were known to societies dating back thousands of years.

We find also that many beliefs without foundation are common. These include misconceptions and prejudices and belief in things for which there is no evidence. Of the latter, astrology is the most widespread. In the unit we examine the historical nature of such beliefs. For example, in ancient Egypt, the year started when the star Sirius was first observed to rise in the morning sky. This was always followed by the flooding of the Nile. Thus they believed the *cause* of the flood was the rising of the star. They then looked for other natural occurrences and related them to star positions. This is an example of the sort of event of the time that led to the belief in astrology. In these situations the Egyptians were reasoning *inductively* in the absence of other information. We know today that Sirius does not cause the Nile to flood and that no other stars have any influence on what happens on earth.

Induction (inductive reasoning) is still useful in establishing beliefs. Everyday examples, generalisations made from observation and patterns might include:

- Red cars go faster
- The days get shorter after Dec. 21st
- February in Brisbane is always hot and humid
- People who live in the country are friendly

Initially we form these beliefs through observation. Later we may learn that there is a logical reason (such as for the changing length of daylight) or that the generalisation is incorrect (You meet an unfriendly country person). The danger of using inductive reasoning to form conclusions is illustrated in tutorial activities (See Appendix 1). Nevertheless, induction is an important in the formation of scientific beliefs. Induction often leads to *hypothesis* formation: For example, the astronomer Kepler observed that the planet Mars moves round the sun in an elliptical orbit. He hypothesised that all planets will do so. This was later observed by

Tyco Brae and subsequently Newton showed (deductively) from his theory of gravitation *why* this must be so, and the statement is now referred to as Kepler’s first law of planetary motion.

The importance of *deductive* reasoning in establishing quantitative beliefs is illustrated by examples such as that the Egyptians (4000 years ago) knew the formulas for areas and volumes, but they didn’t know *why* they worked. For example, they observed that if you made a pyramid of any size, its volume was always 1/3 that of the surrounding prism. The volume of a cone was always 1/3 that of the surrounding cylinder. It was the Greek mathematicians who showed deductively *why* the formulas worked. *Induction and exploration* suggest conclusions which later may be proved. If only the final proof is considered, the process of exploration (including mistakes) is generally lost. Unfortunately much "school" mathematics and science presents only the final results and ignores the procedures that led there. A knowledge of the history of the development helps us understand these procedures and is therefore include in the unit. Tutorial exercises are used to show how inductive and deductive reasoning are used together to arrive at conclusions (See Appendix 1).

Students’ Reflections

Throughout the course, students are asked to respond to the following article which was posted to the OLT (On Line Teaching) Discussion Forum throughout the semester. Responses to this were voluntary.

> Phillip Adams in his column in the Australian Magazine (March 1-2, 2003) states "We could do better if we taught our kids that kicking ideas around is at least as much fun as kicking a football around." To what extent do you believe you have 'kicked' ideas around in this unit so far? Have you been challenged by these ideas? Do you believe your understandings of the nature of mathematics and science has changed? Have you enjoyed the experiences of kicking ideas around? Have your experiences been confined to our tutorial/lecture rooms or are you thinking and talking about these ideas at other times beyond the boundaries of Kelvin Grove campus and timetables. Let the teaching team know your thoughts in this discussion forum.

Earlier research by Peard & Pumadevi (in press) used student responses to this as part of a study into the effectiveness of the unit. This research reports that in Malaysia, where the unit has been taught at two tertiary institutes, as well as at QUT, the implementation of the unit develops better understanding of concepts, and helps develop social skills and most importantly improves attitudes to the subjects. Comments such as “we have enjoyed the experience of kicking ideas around”, “looking at maths and science in this way is very refreshing and exciting and fun” and references to “an open and positive learning environment” `were reported by the authors Peard & Pumadevi (in press) who concluded that the implementation of the unit is consistent with the recommendations of recent research in the field. (See Appendix 2 for a sample of student responses).

Conclusions

The level of quantitative literacy of students entering pre-service primary teacher education is a cause for concern as many students enter these programs with misconceptions about the nature of such knowledge. It is recommended that further research be continued to determine which misconceptions are common and the degree to which they are held. It is also recognized that it is the responsibility of tertiary institutions to make suitable intervention programs within teacher education courses at the tertiary level. Such a program requires the identification of students’ needs in this field and requires the consideration of what constitutes appropriate quantitative knowledge for them. The first objective of any such program must be to break the negativity cycle described in the literature. The conclusion that the program described at QUT is largely successful in doing this is supported by the comments of many students reporting new positive attitudes to the subjects, an appreciation of the importance of quantitative knowledge and improved understanding of the nature of quantitative reasoning.

Appendix 1: Sample Tutorial Activities

1. Draw a circle and connect any two points on the circumference. Two regions are formed. Connect three points to each other and a maximum of 4 regions can be formed. Continue doing this for 4 and 5 points. Make sure each point on the circumference is joined to every other point and complete the table:

No. of points:	2	3	4	5
Maximum no. of regions:	2	4	?	?

Result:

No. of points:	*2*	*3*	*4*	*5*
Maximum no. of regions:	*2*	*4*	*8*	*16*

- Predict how many regions could be formed by 6 points.

Most respond 32

- Test your prediction by drawing a circle and connecting all 6 points to each other.
- What do you notice?

The maximum number of regions is 31

- What is the danger of using inductive reasoning to form conclusions?

2. Polya's "Locker problem"

A school has 1000 numbered lockers, one for each of its students. The students have been asked to line up and walk through the locker area to perform a specific task as follows: Student #1 opens all the locker doors; Student #2 closes each even numbered locker door; Student #3 *reverses* every third locker door (if the locker door is open she closes it; if the locker door is closed she opens it). Each student proceeds through, *reversing* the lockers which correspond to her or his position in line. After all 1000 students have finished, which lockers are open?

By considering the first 20 lockers, we find that the numbers 1, 4, 9, and 16 are open. Inductively we reason that all the square numbered lockers will be open, 31 in all. To prove this however, we must reason deductively.

Appendix 2: Some Sample Responses

- The thought of questioning ideas and process has never really been of concern to me, sure there's been things I've questioned. But, originally when I learn something and that's that.

By questioning and 'kicking around' ideas it opens a new world of learning and I'm quite excited to learn more about it for my sake and for my future students' sake.

- I think that kicking ideas around is a great new way of looking at maths and science. It makes it easier for students like me, whose favourite subjects have never really included maths to get motivated and involved. I like this new way of thinking. I can't wait till I get to use it in a classroom to excite students the same way I've been. Also I think that by kicking ideas around teachers can create a really open and positive learning environment. It can really help with making friends in class.
- I am excited by the idea of approaching maths and science with a more flexible attitude. All of my previous forays into this field have seemed rigid and with no room for error. I'm looking forward to learning how to kick ideas around and approach problems logically, not just by recalling equations learnt by rote.
- Although I'm not a fan of maths and science, and was feeling intimidated at first, (still a little shaky) I am loving this unit. Being able to discuss problems with a group in an environment solely set on encouraging higher thinking is great. And yes I have had a few light bulb moments!
- I graduated 20 years ago! So I was a child of the rote learning method of everything. Memorising formulas and all that jazz. On reflection I can see now why I did well in maths in some years and performed poorly in others. In the years I performed poorly I was not enjoying the content because the teachers did not convey its relevance and meaning and I could not relate anything to 'real life'.
- These first lectures and the first tut made everything seem interesting and challenging again. I enjoyed the group interaction because everyone in our group brought different strengths and skills to the table - a sharing experience that was missing from my school years.
- I love the tutorials, and the lectures as they open my mind up and challenge it to look further than what I have been using in years, and the last lecture when I got told that I didnt need to learn all those formulas I knew during high school I was wasting my time in maths for four years memorising those formulas.

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